Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Specimen Paper

(Time: 1 hour 30 minutes) Paper Reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Lilac), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

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Answer ALL questions. Write your answers in the spaces provided.

1. (a) Find the first four terms, in ascending powers of x, in the binomial expansion of

$$\left(1 - \frac{1}{8}x\right)^{10}$$

giving each term in its simplest form.

(3)

(b) Hence find the coefficient of x^3 in the expansion of

$$(3 + 10x)\left(1 - \frac{1}{8}x\right)^{10}$$

(2)

(a) Binomial expansion formula :

$$(a+bx)^n = {}^nC_0\, a^n (bx)^0 + {}^nC_1\, a^{n-1}(bx)^1 + {}^nC_2\, a^{n-2}(bx)^2 + \dots$$

$${}^nC_r = \frac{n!}{(n-r)!\,r!}$$

n: power of binomial

r: (position of term) -1

OR: USE CHOOSE FUNCTION ON CALCULATOR

for $\left(1 - \frac{1}{8}x\right)^{10}$:

$$= {}^{10}C_0\, 1^{10}\left(\frac{-x}{8}\right)^0 + {}^{10}C_1\, 1^{10-1}\left(\frac{-x}{8}\right)^1 + {}^{10}C_2\, 1^{10-2}\left(\frac{-x}{8}\right)^2 + {}^{10}C_3\, 1^{10-3}\left(\frac{-x}{8}\right)^3$$

${}^{10}C_0 = 1$

${}^{10}C_1 = 10$

${}^{10}C_2 = \frac{10!}{(10-2)!\,2!} = 45$

${}^{10}C_3 = \frac{10!}{(10-3)!\,3!} = 120$

NOTES:-
- the first and last ${}^nC_r$ are always $=1$
- ${}^nC_1$ is always equal to n

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Question 1 continued

$= (1)(1)(0) + 10(1)\left(\frac{-x}{8}\right) + 45(1)\left(\frac{x^2}{64}\right) + 120(1)\left(\frac{-x^3}{512}\right)$

$= 1 - \frac{5}{4}x + \frac{45}{64}x^2 - \frac{15}{64}x^3$

(b) $(3+10x)\left(1 - \frac{1}{8}x\right)^{10}$

$= (3+10x)\left(1 - \frac{5}{4}x + \frac{45}{64}x^2 - \frac{15}{64}x^3\right)$

↳ possible terms with term x^3

① $10x \times \frac{45}{64}x^2 = \frac{225}{32}x^3$

② $3 \times \frac{-15}{64}x^3 = \frac{-45}{64}x^3$

So: $\frac{225}{32}x^3 + \frac{-45}{64}x^3 = \frac{405}{64}x^3$

coefficient $= \frac{405}{64}$

Q1

(Total for Question 1 is 5 marks)

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2.

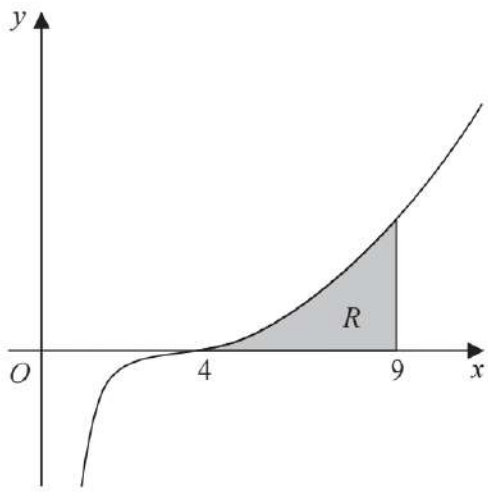

Figure 1

In this question you must show all steps of your working.

Solutions relying on calculator technology are not acceptable.

A curve has equation

$$y = 2x\sqrt{x} - \frac{16}{x^2} - 6x + 9 \qquad x > 0$$

The curve crosses the x-axis at the point (4, 0)

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line with equation $x = 9$

Find the exact area of R.

(6)

① rewrite equation :

$$y = 2x\sqrt{x} - \frac{16}{x^2} - 6x + 9$$

$$\boxed{\text{use : } \sqrt[a]{b} = b^{1/a}}$$

$$y = 2x(x^{1/2}) - \frac{16}{x^2} - 6x + 9$$

Question 2 continued

$$\boxed{\text{use : } \frac{1}{a^b} = a^{-b}}$$

$$y = 2x(x^{1/2}) - 16x^{-2} - 6x + 9$$

$$\boxed{\text{use : } x^a \times x^b = x^{a+b}}$$

$$y = 2x^{1+1/2} - 16x^{-2} - 6x + 9$$

$$y = 2x^{3/2} - 16x^{-2} - 6x + 9$$

now, we can integrate this!

REMEMBER!

$$\boxed{\int x^n\,dx = \frac{x^{n+1}}{n+1}}$$

$$y = 2x^{3/2} - 16x^{-2} - 6x + 9$$

$$\int 2x^{3/2} - 16x^{-2} - 6x + 9 \, dx$$

$$= \frac{2x^{3/2+1}}{3/2+1} - \frac{16x^{-2+1}}{-2+1} - \frac{6x^{1+1}}{1+1} + \frac{9x^{0+1}}{0+1}$$

(Total for Question 2 is 6 marks)

Q2

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$$= \frac{2x^{5/2}}{5/2} - \frac{16x^{-1}}{-1} - \frac{6x^2}{2} + \frac{9x^1}{1}$$

$$= \frac{4}{5}x^{5/2} + \frac{16}{x} - 3x^2 + 9x \quad (+c)$$

The area (R) is bounded by : $x = 4$, $x = 9$ (see diagram). So,

$$A_R = \int_4^9 2x^{3/2} - 16x^{-2} - 6x + 9 \quad dx$$

$$= \left. \frac{4}{5}x^{5/2} + \frac{16}{x} - 3x^2 + 9x \right]_4^9$$

$$= \left(\frac{4}{5}(9)^{5/2} + \frac{16}{9} - 3(9)^2 + 9(9)\right) - \left(\frac{4}{5}(4)^{5/2} + \frac{16}{4} - 3(4)^2 + 9(4)\right)$$

$$= \frac{1538}{45} - \frac{88}{5}$$

$$= \frac{746}{45} \quad \text{(exact)}$$

(note : In MS, the final answer is given as a mixed number = $16\ ^{26}/_{45}$)

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3. In an arithmetic series

- the sixth term is 11 $a_6 = 11$
- the sum of the first three terms is –15 $S_3 = -15$

↳ partial sum

For this series,

(a) show that the first term is –9 and calculate the common difference. **(4)**

Given that $S_n > 888$

(b) find the least value of n. **(3)**

(a) We know that for an arithmetic series :

$a_n = a + d(n-1)$

$a_6 = a + d(6-1)$

① $11 = a + d(6-1)$ ⟶ equation 1

Use partial sum formula :

$S_N = \frac{N}{2}(2a + (N-1)d)$

$S_3 = \frac{3}{2}(2a + (3-1)d)$

② $-15 = \frac{3}{2}(2a + 2d)$ ⟶ equation 2

so now we can solve the two equations simultaneously :

(1) $a + 5d = 11$

(2) $\frac{3}{2}(2a + 2d) = -15$

Question 3 continued

(1) Elimination :

$a + 5d = 11$

$\frac{3}{2}(2a + 2d) = -15$

$(a + 5d = 11) \times -3$

$3a + 3d = -15$

$-3a - 15d = -33$

$+ (3a + 3d = -15)$

$-12d = -48$

$d = 4$

$a + 5(4) = 11$

$a = -9$

(2) Substitution :

$a + 5d = 11$

$a = 11 - 5d$

$\frac{3}{2}(2a + 2d) = -15$

$\frac{3}{2}(2(11 - 5d) + 2d) = -15$

$\frac{3}{2}(22 - 10d + 2d) = -15$

$[\frac{3}{2}(22 - 8d) = 15] \times \frac{2}{3}$

$22 - 8d = -10$

$8d = 22 + 10$

$8d = 32$

$d = 4$

$a = 11 - 5d$

$= 11 - 5(4)$

$a = -9$

(b) Use partial sum formula :

$S_n = \frac{n}{2}(2a + (n-1)d)$

Q3

(Total for Question 3 is 7 marks)

$$S_n = \frac{n}{2}\left(2(-9) + (n-1)(4)\right) > 888$$

$$\frac{n}{2}\left(2(-9) + (n-1)(4)\right) > 888$$

$$\frac{n}{2}\left(-18 + 4n - 4\right) > 888$$

$$-9n + 2n^2 - 2n > 888$$

$$2n^2 - 11n > 888$$

$2n^2 - 11n - 888 > 0 \longrightarrow$ FIND ROOTS:

$$2n^2 - 11n - 888 = 0$$

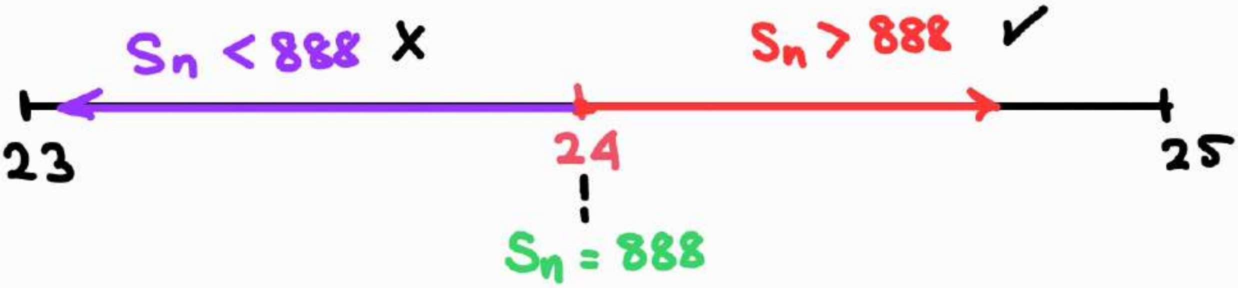
$$(2n + 37)(n - 24) = 0$$

$$n = 24$$

or $n = -\frac{37}{2}$ $\rightarrow$ reject as n can't be negative or a fraction!

so : when $n = 24$;

$$2n^2 - 11n = 888$$



so, round UP!, $n = 25$

4.

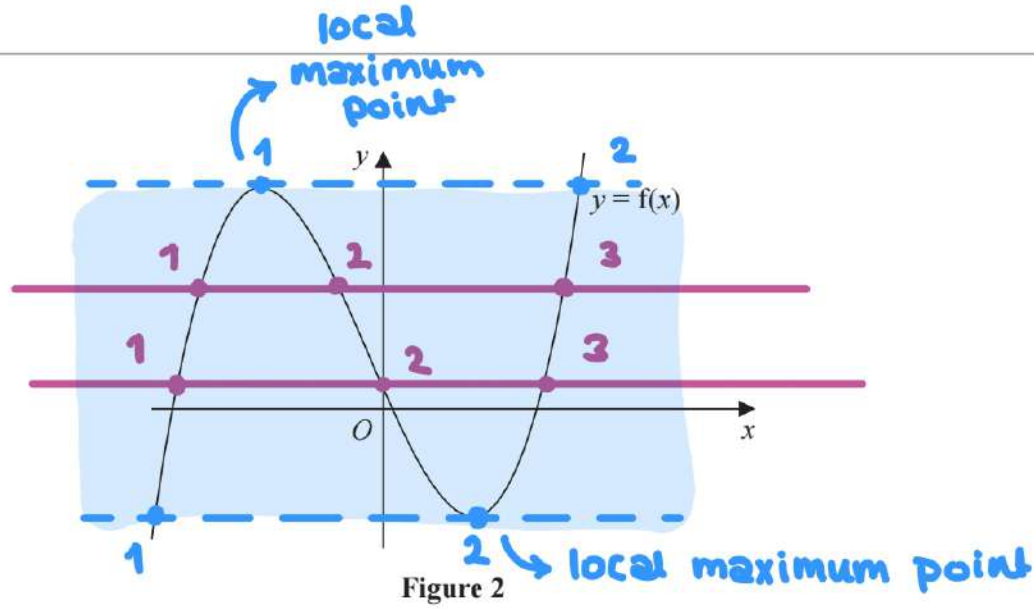


Figure 2

In this question you must show all steps of your working.

Solutions relying on calculator technology are not acceptable.

Figure 2 shows a sketch of the curve with equation $y = f(x)$ where

$$f(x) = 2x^3 + \frac{3}{2}x^2 - 18x + 3$$

(a) Find the set of values of x for which $f(x)$ is decreasing.

(4)

Given that the equation $f(x) = k$, where k is a constant, has one real root,

(b) find the set of values for k.

(3)

(a) When $f(x)$ is decreasing, $f'(x) < 0$.
(negative gradient).

$$f(x) = 2x^3 + \frac{3}{2}x^2 - 18x + 3$$

DIFFERENTIATION

$f(x) = ax \longrightarrow f'(x) = nax^{n-1}$

$f'(x) = (3)2x^{3-1} + (2)\frac{3}{2}x^{2-1} - (1)18x + (0)3x^{0-1}$

$f'(x) = 6x^2 + 3x - 18x$

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Question 4 continued

Find $f'(x) = 0$, so you can determine the zone where $f'(x) < 0$.

$$f'(x) = 0$$
$$6x^2 + 3x - 18 = 0$$
$$\tfrac{1}{3} \times (6x^2 + 3x - 18) = 0 \times \tfrac{1}{3}$$
$$2x^2 + x - 6 = 0$$

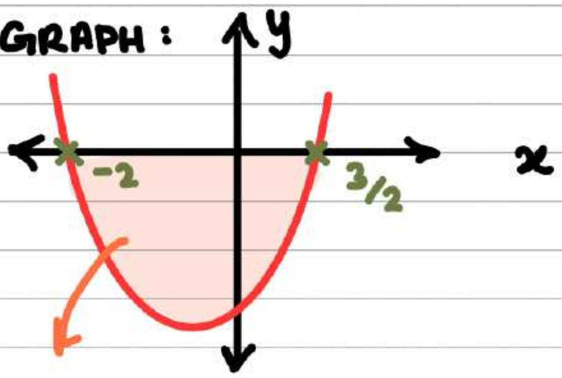
FACTORISE :

$$(x+2)(2x-3) = 0$$

SO : $x + 2 = 0$ OR $2x - 3 = 0$

$x < -2$ $\qquad$ $x < \frac{3}{2}$

IMAGINE GRAPH :



graph is negative between -2 and $3/2$, so:

$$-2 < x < 3/2$$

(b) LOOK AT FIG 2 :

In the blue region, any straight line $y = k$ will give 3 solutions to $f(x) = k$.

At the local maximum and minimum points, $f(x) = k$ has 2 solutions.

Q4

(Total for Question 4 is 7 marks)

Where $f(x) = k$ has only one solution, k is outside the blue region.

① Find local maximum and minimum:

At max and min points, $f'(x) = 0$

we found in part (a) that at $f'(x)=0$,

$$x = -2 \quad \text{or} \quad x = 3/2$$

$$f(-2) = 2(-2)^3 + \frac{3}{2}(-2)^2 - 18(-2) + 3 = 29$$

$$f(3/2) = 2\left(\frac{3}{2}\right)^3 + \frac{3}{2}\left(\frac{3}{2}\right)^2 - 18\left(\frac{3}{2}\right) + 3 = -13.875$$

so: $k < -13.875$ and $k > 29$

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5. $$f(x) = x^3 - 8x + a$$

When $f(x)$ is divided by $(x + 2)$ the remainder is -24

(a) Use the remainder theorem to show that $a = -32$

(2)

Given also that $(x - 4)$ is a factor of $f(x)$

(b) (i) write $f(x)$ as a product of two algebraic factors,

(ii) prove that the equation $f(x) = 0$ has only one real root.

(4)

(a)

use : REMAINDER THEOREM :

if $\frac{f(x)}{(x-k)}$ = quotient + remainder :

then $f(k)$ = remainder

if $\frac{f(x)}{(x-(-2))}$ = quotient + (-24)

then $f(-2) = -24$

↳ plug $x = -2$ into $f(x)$ to find a :

$f(x) = x^3 - 8x + a$

$f(-2) = (-2)^3 - 8(-2) + a = -24$

$-8 + 16 + a = -24$

$8 + a = -24$

$a = -24 - 8$

$a = -32$

(b) (i) Use algebraic division to factorise $f(x)$:

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Question 5 continued

$$x-4 \overline{\big) \, x^3 - 8x - 32}$$

(1) Divide first term of dividend by first term of divisor : $\frac{x^3}{x} = x^2 \rightarrow$ first term of quotient

$$\begin{array}{r} x^2 \\ x-4 \overline{\big) \, x^3 - 8x^2 - 32} \end{array}$$

$\div$

(2) Multiply out and write under :

$$\begin{array}{r} x^2 \\ x-4 \overline{\big) \, x^3 - 8x - 32} \\ \underline{x^3 - 4x^2 } \end{array}$$

REMEMBER NOT TO SUBTRACT A TERM WITH x^2 FROM TERM WITH x.

(3) Subtract then bring down next term :

$$\begin{array}{r} x^2 \\ x-4 \overline{\big) \, x^3 - 8x - 32} \\ \underline{-(x^3 - 4x^2) } \\ 4x^2 - 8x^0 + 19x \end{array}$$

(4) Divide first term of expression by first term of divisor : $\frac{4x^2}{x} = 4x \rightarrow$ next term of quotient

$$\begin{array}{r} x^2 + 4x \\ x-4 \overline{\big) \, x^3 - 8x - 32} \\ \underline{-(x^3 - 4x^2) } \\ 4x^2 - 8x - 32 \end{array}$$

Q5

(Total for Question 5 is 6 marks)

(5) Multiply out and write under:

$$
\begin{array}{r|l}
 & \quad x^2+4x \\ \cline{2-2}
x-4 & x^3-8x-32 \\
 & -(x^3-4x^2) \\ \cline{2-2}
 & \qquad 4x^2-8x-32 \\
 & \qquad 4x^2-16x \\ \cline{2-2}
\end{array}
$$

(6) Subtract then bring down next term:

$$
\begin{array}{r|l}
 & \quad x^2+4x \\ \cline{2-2}
x-4 & x^3-8x-32 \\
 & -(x^3-4x^2) \\ \cline{2-2}
 & \qquad 4x^2-8x-32 \\
 & \quad -(4x^2-16x) \downarrow \\ \cline{2-2}
 & \qquad\qquad 8x-32
\end{array}
$$

(7) Divide first term of expression by first term of divisor: $\dfrac{8x}{x} = 8$ → next term of quotient

$$
\begin{array}{r|l}
 & \quad x^2+4x+8 \\ \cline{2-2}
x-4 & x^3-8x-32 \\
 & -(x^3-4x^2) \\ \cline{2-2}
 & \qquad 4x^2-8x-32 \\
 & \quad -(4x^2-16x) \\ \cline{2-2}
 & \qquad\qquad 8x-32
\end{array}
$$

(8) Multiply out and write under:

$$
\begin{array}{r|l}
 & \quad x^2+4x+8 \\ \cline{2-2}
x-4 & x^3-8x-32 \\
 & -(x^3-4x^2) \\ \cline{2-2}
 & \qquad 4x^2-8x-32 \\
 & \quad -(4x^2-16x) \\ \cline{2-2}
 & \qquad\qquad 8x-32 \\
 & \qquad\qquad 8x-32
\end{array}
$$

(9) Subtract

$$
\begin{array}{r|l}
 & \quad x^2 + 4x + 8 \\
\hline
x-4 & x^3 - 8x - 32 \\
 & -(x^3 - 4x^2) \\
\hline
 & \qquad 4x^2 - 8x - 32 \\
 & \qquad -(4x^2 - 16x) \\
\hline
 & \qquad\qquad 8x - 32 \\
 & \qquad\qquad -(8x - 32) \\
\hline
 & \qquad\qquad\qquad 0
\end{array}
$$

So: $f(x) = (x-4)(x^2+4x+8)$.

(ii) We know our real root is $x=4$.

Prove that $(1x^2 + 4x + 8)$ has <u>no</u> real roots:

REMEMBER:

$$x = \frac{-b \pm \sqrt{\boxed{b^2 - 4ac}}}{2a}$$

discriminant

If discriminant >0, then eq. has two solutions
If discriminant $=0$, then eq. has one solution
If discriminant <0, then eq. has no solutions

$b^2 - 4ac$
$= (4)^2 - 4(1)(8)$
$= -16 < 0$ so x^2+4x+8 has NO ROOTS

So, $f(x)$ has only one real root!

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6. (a) Sketch the curve with equation

$$y = a^x, \quad a > 1$$

showing the coordinates of any points of intersection with the coordinate axes. **(2)**

Interval btwn two x-coords = h of trapezium

x	2	2.4	2.8	3.2	3.6	4
y	11 (y_1)	16.37	24.47	36.83	55.80	85 (y_6)

multiplied by 2 as all are shared by two trapezia

The table above shows corresponding values of x and y for $y = 3^x + x$. The values of y are given to two decimal places as appropriate.

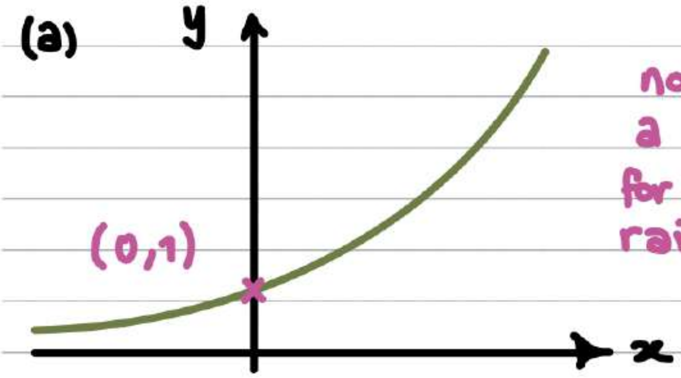
Using the trapezium rule with all the values of y in the given table,

(b) obtain an estimate for $\int_2^4 (3^x + x)\,dx$, giving your answer to one decimal place. **(3)**

Using your answer to part (b) and making your method clear, estimate

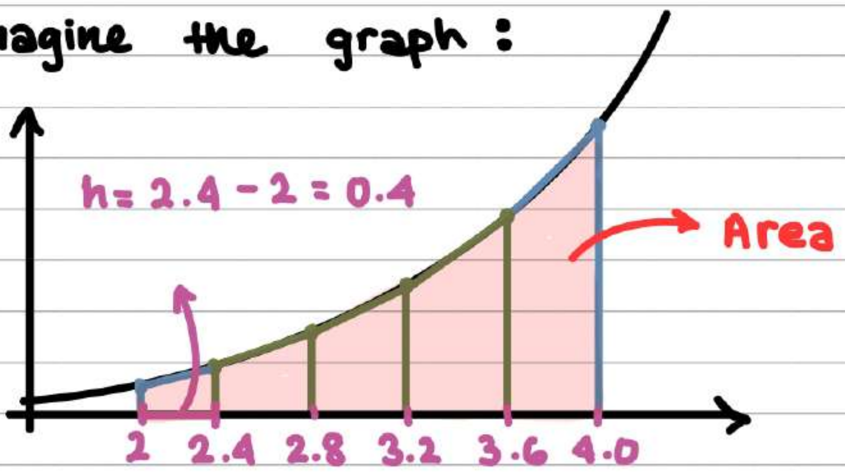
(c) $\int_2^4 (3^{x+1} + x)\,dx$ **(3)**

(a)



note : (0,1) is a common point for any constant raised to the power of x.

(b) Imagine the graph :



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Question 6 continued

(Formula is given in formula booklet. The graph is only a visual representation. DO NOT DRAW A GRAPH IN THE EXAM)

$$A = \frac{1}{2}h\left[y_1 + 2(y_2 + y_3 + y_4 + y_5) + y_6\right]$$

$$= \frac{1}{2}(0.4)\left[11 + 2(16.37 + 24.47 + 36.83 + 55.80) + 85\right]$$

$$= 0.2(362.88)$$

$$= 72.6 \text{ (1 d.p.)}$$

(c) $\int_2^4 (3^{x+1} + x)\, dx$

use rule: $a^{b+c} = a^b \times a^c$

$$= \int_2^4 3^x \times 3^1 + x \, dx$$

$$= \int_2^4 3 \times 3^x + (3x - 2x) \, dx$$

$$= \int_2^4 3(3^x) + 3x - 2x \quad dx$$ → take 3 as common factor

$$= \int_2^4 3(3^x + x) - 2x \quad dx$$

USE RULE:

$$\int f(x) + g(x)\, dx = \int f(x)\, dx + \int g(x)\, dx$$

Q6

(Total for Question 6 is 8 marks)

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$$= \int_2^4 3(3^x + x) - 2x \quad dx$$

$$= \int_2^4 3(3^x + x) \; dx - \int_2^4 2x \; dx$$

$$= \int_2^4 3(3^x + x) \; dx - \int_2^4 2x \; dx$$

use rule : $\int a f(x) \, dx = a \int f(x) \, dx$

$$= 3 \boxed{\int_2^4 3^x + x \quad dx} - \boxed{\int_2^4 2x} \, dx$$

value estimated in (b).

integrate this!

REMEMBER!

$$\boxed{\int x^n dx = \frac{x^{n+1}}{n+1}}$$

$$= 3(72.6) - \left[\frac{2x^{1+1}}{1+1} \right]_2^4$$

$$= 217.8 - \left[x^2 \right]_2^4$$

$$= 217.8 - (4^2 - 2^2)$$

$$= 217.8 - 12$$

$$= 205.8$$ → FINAL ANSWER!

7. (i) Find, in terms of $\log_2 3$, an exact solution of the equation

$$3 \times 2^{x-2} = 8^x$$

(4)

(ii) Solve, using algebra, the equation

$$2\log_5(2y+1) - \log_5(2-y) = 1$$

explaining clearly why there is only one solution.

(4)

(i) $3 \times 2^{x-2} = 8^x$

(1) Rewrite 8 as $8 = 2^3$:

$$3 \times 2^{x-2} = (2^3)^x$$

(2) Use rule : $(a^b)^c = a^{b \times c}$

$$3 \times 2^{x-2} = 2^{3x}$$

(3) Divide both sides by 2^{x-2}

$$\frac{3 \times 2^{x-2}}{2^{x-2}} = \frac{2^{3x}}{2^{x-2}}$$

$$\frac{2^{3x}}{2^{x-2}} = 3$$

(4) Use rule : $\frac{a^b}{a^c} = a^{b-c}$

$$2^{3x-(x-2)} = 3$$

$$2^{3x-x+2} = 3$$

$$2^{2x+2} = 3$$

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Question 7 continued

(5) take $\log_2$ of both sides :

$$\log_2 2^{2x+2} = \log_2 3$$

(6) use rule : $\log a^b = b\log a$

$$\log_2 2^{2x+2} = \log_2 3$$

$$(2x+2)\log_2 2 = \log_2 3$$

(7) $\log_a a = 1$ so $\log_2 2 = 1$:

$$(2x+2)(1) = \log_2 3$$

$$2x+2 = \log_2 3$$

$$\frac{2x}{2} = \frac{\log_2 3 - 2}{2}$$

$$\therefore \ x = \frac{\log_2 3}{2} - 1$$

ALTERNATIVE SOLUTION :

(1) take $\log_2$ of both sides :

$$3 \times 2^{x-2} = 8^x$$

$$\log_2 3 \times 2^{x-2} = \log_2 8^x$$

(2) $\log a \times b = \log a + \log b$

$$\log_2 3 + \log_2 2^{x-2} = \log_2 8^x \quad = 2^{3x}$$

Q7

(Total for Question 7 is 8 marks)

(3) $\log a^b = b\log a$

$$\log_2 3 + \log_2 2^{x-2} = \log_2 2^{3x}$$

$$\log_2 3 + (x-2)\log_2 2 = 3x\log_2 2 \rightarrow \log_a a = 1$$

$$\log_2 3 + x - 2 = 3x$$

$$\frac{2x}{2} = \frac{\log_2 3 - 2}{2}$$

$$\therefore \quad x = \frac{\log_2 3}{2} - 1$$

(ii) Recall rules of logarithms :

$$\log_a(b) + \log_a(c) = \log_a(b \times c)$$
$$\log_a(b) - \log_a(c) = \log_a\left(\frac{b}{c}\right)$$
$$\log a^b = b\log a$$

$$2\log_5(2y+1) - \log_5(2-y) = 1$$

$$\log_5(2y+1)^2 - \log_5(2-y) = 1$$

$$\log_5\left(\frac{(2y+1)^2}{2-y}\right) = 1$$

RECALL!

$$\log_a x = b$$
$$a^b = x$$

expand!

$$\frac{(2y+1)^2}{2-y} = 5^1$$

$$(2-y) \times \left(\frac{4y^2 + 4y + 1}{2-y}\right) = (5) \times (2-y)$$

$$4y^2 + 4y + 1 = 5(2-y)$$

$$4y^2 + 4y + 1 = 10 - 5y$$

$$4y^2 + 9y - 9 = 0$$ → FACTORISE :

$$(4y-3)(y+3) = 0$$

$y = \frac{3}{4}$ or $y = -3$ × REJECT

$y = -3$ is not a solution of the eq :

$$2\log_5(2(-3)+1) - \log_5(2-(-3)) = 1$$

$$2\log_5(-5) - \log_5(5) = 1$$

→ log of a negative is not possible!

$$\boxed{\text{so } y = 3/4}$$

!!!

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8. The depth of water, D metres, in a harbour on a particular day is modelled by the equation

$$D = 3 - 1.2\sin\left(\frac{\pi t}{6}\right), \qquad 0 \leqslant t < 19$$

!!

where t is the number of hours after 05:00, and $\frac{\pi t}{6}$ is measured in radians.

YOUR CALCULATOR SHOULD BE IN RADIANS!

(a) Show that the depth of the water at 06:00 is 2.4 metres.
(1)

(b) State the greatest value of D.
(1)

(c) Find the first two times of the day, after 05:00, when the depth of the water in the harbour is 3.5 metres. Show all steps in your working and give your answers to the nearest minute.
(6)

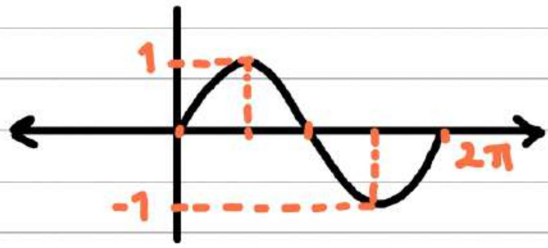
(a) 06 : 00 is 1 hour after 05 : 00

Substitute $t = 1$ into given eq. :

$$D = 3 - 1.2\sin\left(\frac{\pi(1)}{6}\right)$$

$D = 2.4$ m

(b) Think of the graph of $\sin\theta$:



For $D = 3 - 1.2\sin(\theta)$, the maximum D is at minimum $\sin\theta$

range of $\sin\theta \rightarrow -1 \leq \sin\theta \leq 1$

so minimum $\sin\theta = -1$

$D_{max} = 3 - 1.2(-1) = 3 + 1.2 = 4.2$

Leave blank

Question 8 continued

(c) Substitute $H = 3.5$

$$3.5 = 3 - 1.2\sin\left(\frac{\pi t}{6}\right)$$

$$3.5 - 3 = -1.2\sin\left(\frac{\pi t}{6}\right)$$

$$\frac{1}{-1.2} \times (0.5) = \left(-1.2\sin\left(\frac{\pi t}{6}\right)\right) \times \frac{1}{-1.2}$$

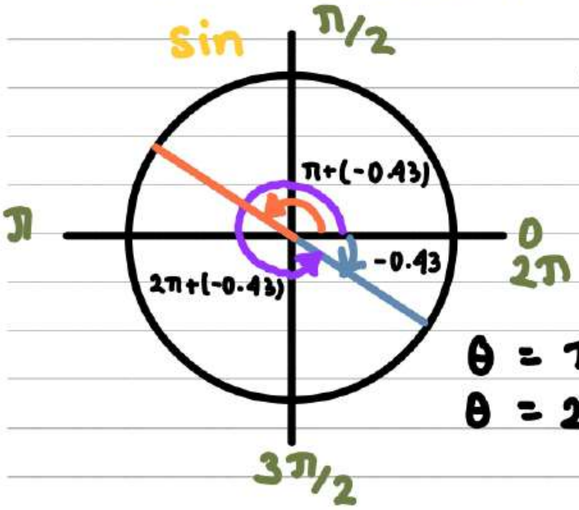
$$\frac{-5}{12} = \sin\left(\frac{\pi t}{6}\right)$$

$$\frac{\pi t}{6} = \sin^{-1}\left(\frac{-5}{12}\right)$$

FROM CALCULATOR:

$$\frac{\pi t}{6} = -0.429775\ldots$$

ON UNIT CIRCLE:



$$\sin\left(\frac{-5}{12}\right) = -0.4297$$

ALSO:

$$\sin(-\theta) = -\sin\theta$$

SO:

$$\theta = \pi + (-0.4297) = 3.5713\ldots$$
$$\theta = 2\pi + (-0.4297) = 5.8534\ldots$$

$$6\times\left(\frac{\pi t}{6}\right) = \underset{\times 6}{(-0.4297)}, \underset{\times 6}{(3.5713)}, \underset{\times 6}{(5.8534)}$$

Q8

(Total for Question 8 is 8 marks)

S 6 3 0 1 5 A 0 1 7 2 4

$$\frac{\pi t}{\pi} = \frac{-2.5782}{\pi}, \frac{21.4278}{\pi}, \frac{35.1204}{\pi}$$

$t = -0.820$, 6.82, 11.18

× reject, time should not be negative!

↳ 6.82 hours = 6 hours + 0.82(60) mins

6 hours, 49.2 minutes

= 6 hrs 49 mins PAST 5 (nearest minute).

so 5AM + 6hrs 49 mins

= 11:49 AM

11.18 hours = 11 hours + 0.18(60) mins

11 hours, 10.8 minutes

= 11 hrs 11 mins PAST 5 (nearest minute).

so 5 AM + 11 hrs 11 mins

= 4:11 PM

Leave blank

9. The first three terms of a geometric series are

$\sin\theta$, $\cos\theta$ and 0.5 where θ is a constant

a_1 a_2 a_3

(a) Show that

$$2\sin^2\theta + \sin\theta - 2 = 0$$

(3)

Given that θ lies in the interval $90° < \theta < 180°$

(b) find the value of θ giving your answer to one decimal place.

(4)

(c) Hence prove that this series is convergent.

(3)

(d) Find, to two significant figures, S_∞

(2)

(a) In a geometric series :

$$\underset{\sin\theta}{a_1} \xrightarrow{\times r} \underset{\cos\theta}{a_2} \xrightarrow{\times r} \underset{0.5}{a_3}$$

Where : $r = \dfrac{a_2}{a_1} = \dfrac{a_3}{a_2}$

$$\frac{\cos\theta}{\sin\theta} = \frac{0.5}{\cos\theta}$$ CROSS MULTIPLY

$$\cos^2\theta = 0.5\sin\theta$$

USE IDENTITY :

$$\sin^2\theta + \cos^2\theta = 1$$
$$\cos^2\theta = 1 - \sin^2\theta$$

$$1 - \sin^2\theta = 0.5\sin\theta$$

$$2 \times (\sin^2\theta + 0.5\sin\theta - 1) = (0) \times 2$$

Question 9 continued

$$2\sin^2\theta + \sin\theta - 2 = 0$$

(b) $2\sin^2\theta + 1\sin\theta - 2 = 0$

$2x^2 + x - 2 = 0$

You can use quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sin\theta = \frac{-1 \pm \sqrt{1^2 - 4(2)(-2)}}{2(2)}$$

$$\sin\theta = \frac{-1 + \sqrt{17}}{4} \quad (\approx 0.780)$$

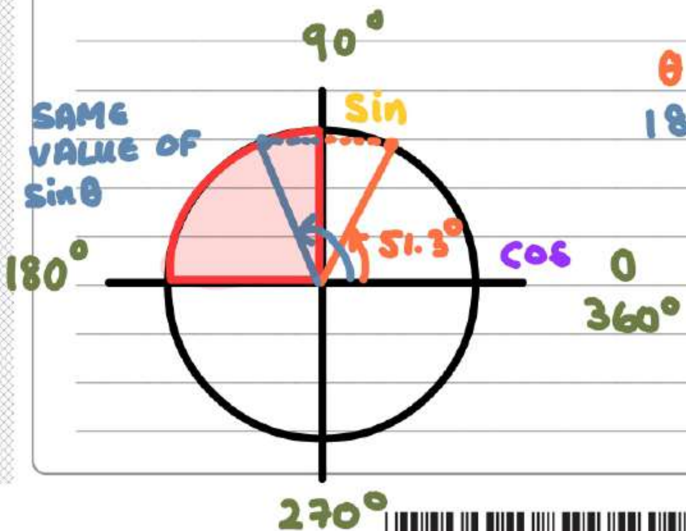
$$\sin\theta = \frac{-1 - \sqrt{17}}{4} \quad (\approx -1.28)$$ → REJECT; range of $f(x) = \sin\theta$ is $-1 \leq \sin\theta \leq 1$!

So: $\sin\theta = \dfrac{-1 + \sqrt{17}}{4}$

ON CALCULATOR:

$$\theta = \arcsin\left(\frac{-1+\sqrt{17}}{4}\right) = \underline{51.33^\circ}$$ not within domain !!

Since our domain is $90^\circ < \theta < 180^\circ$:



$\theta = 51.3$

$180 - 51.3 = 128.7^\circ$

So $\theta = 128.7^\circ$

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Question 9 continued

(c) A geometric series will converge ONLY

if : $-1 < r < 1$

we know $r = \frac{\cos\theta}{\sin\theta}$,

$r = \frac{\cos(128.7°)}{\sin(128.7°)} = -0.801...$

$-1 < -0.801 < 1$, so the series converges!

(d) Use sum to infinity formula:

$S_\infty = \frac{a}{1-r}$

$S_\infty = \frac{\sin(128.7°)}{1-(-0.801)} = 0.433...$

$= 0.43$ (2 s.f.)

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Question 9 continued

(Total for Question 9 is 12 marks)

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Q9

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10.

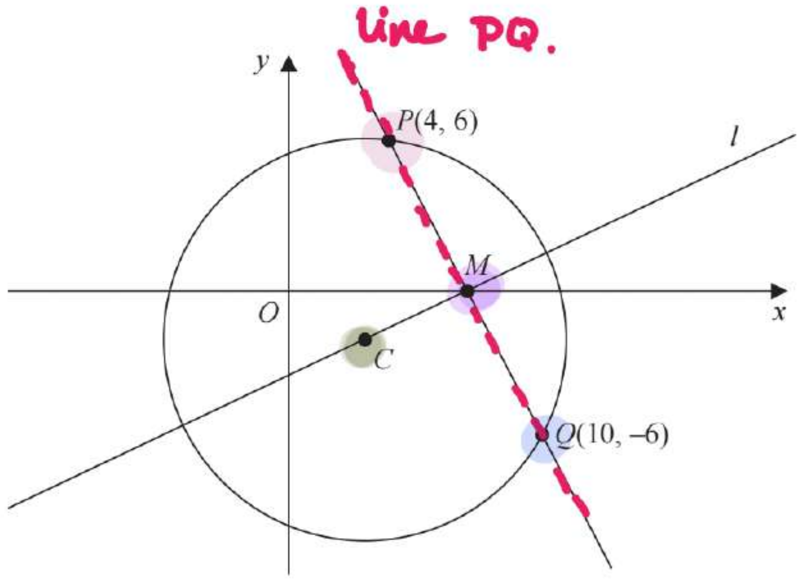

Figure 3

Figure 3 shows a circle with centre C.

The point $P(4, 6)$ and the point $Q(10, -6)$ both lie on the circumference of the circle.

The point M is the midpoint of PQ.

The line l passes through C and M.

(a) Find an equation for l, giving your answer in the form $px + qy + r = 0$, where p, q and r are integers to be found.

(4)

Given that the y coordinate of C is -2

(b) find an equation for the circle.

(4)

(a)- M is midpoint of PQ, so:

$$\text{coords}_M = \left(\frac{x_P + x_Q}{2}, \frac{y_P + y_Q}{2}\right)$$

$$= \left(\frac{4+10}{2}, \frac{6+(-6)}{2}\right)$$

$$M = (7, 0)$$

$$y = mx + c$$

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Question 10 continued

$$m_{PQ} = \frac{y_P - y_Q}{x_P - x_Q} = \frac{6-(-6)}{4-10} = -2$$

$$m_l = \frac{-1}{m_{PQ}} = \frac{-1}{-2} = \frac{1}{2}$$

$$y = \frac{1}{2}x + c$$

METHOD 1 :

FIND c, use $(7,0)$:

$$y = mx + c$$

$$0 = \frac{1}{2}(7) + c$$

$$c = -\frac{7}{2}$$

$$2\times(y) = \left(\frac{1}{2}x - \frac{7}{2}\right)\times 2$$

$$2y = x - 7$$

$$x - 2y - 7 = 0$$

METHOD 2 :

use $(y - y_1) = m(x - x_1)$

$$y - 0 = \frac{1}{2}(x - 7)$$

$$2\times(y) = \left(\frac{1}{2}x - \frac{7}{2}\right)\times 2$$

$$2y = x - 7$$

$$x - 2y - 7 = 0$$

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Question 10 continued

(b) Eq. of circle :

$(x - x_{centre})^2 + (y - y_{centre})^2 = r^2$

c is also a point on line l, so substitute $y = -2$ into line l to find x_c :

$$y_c = \tfrac{1}{2}x_c - \tfrac{7}{2}$$

$$-2 = \tfrac{1}{2}x_c - \tfrac{7}{2}$$

$$\frac{1}{\cancel{2}}x_c = \frac{3}{\cancel{2}}$$

$$x_c = 3$$

coords C = (3, -2)

x, y are any point on circle C :

use P(4, 6) to find r^2 :

$(x - x_{centre})^2 + (y - y_{centre})^2 = r^2$

$(4 - 3)^2 + (6 - (-2))^2 = 65$

so eq. of circle:

$(x - 3)^2 + (y + 2)^2 = 65$

Q10

(Total for Question 10 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS

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